

# Journées $SL_2\mathbb{R}$

Institut Élie Cartan de Lorraine (UMR CNRS 7502)

Université de Lorraine

June 9–10, 2016

## Conferences:

Amphi Hermite, Bâtiment de l'UFR MIM  
Ile du Saulcy, Université de Lorraine, Metz

## Program

### Thursday, June 9

- 13:40–13:45 *Welcome and Opening Remarks*
- 13:45–14:35 Toshiyuki KOBAYASHI (University of Tokyo) :  
*Branching problems and symmetry breaking operators*
- 14:45–15:40 Michael PEVZNER (Université de Reims) :  
*Symmetry breaking operators for differential forms*
- 15:45–16:20 *Coffee Break*
- 16:20–17:15 Martin OLBRICH (University of Luxembourg) :  
*TBA*
- 17:20–18:15 Pavle PANDZIC (University of Zagreb) :  
*A generalization of Dirac index*
- 19:30 *Dinner:*

### Friday, June 10

- 8:30–9:25 Michèle VERGNE (Université Paris VII) :  
*Indice équivariant des opérateurs de Dirac*
- 9:30–10:25 Roger ZIERAU (Oklahoma State University) :  
*Characteristic cycles of highest weight Harish-Chandra modules*
- 10:30–11:00 *Coffee Break*
- 11:00–11:55 Genkai ZHANG (Chalmers and Göteborg University) :  
*Boundary value problem for some conformal invariant differential operators*
- 12:00–12:55 Tomasz PRZEBINDA (University of Oklahoma) :  
*The character and the wave front set correspondence in the stable range*
- 13:00–14:30 *Lunch Break*
- 14:30–15:25 Michel DUFLO (Université Paris VII) :  
*On the restriction of discrete series of a real reductive Lie group to a subgroup locally isomorphic to  $SL(2, \mathbb{R})$*
- 15:30–16:25 David VOGAN (Massachusetts Institute of Technology) :  
*Langland parameters and finite-dimensional representations*

# Abstracts

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**Michel DUFLO:**

*On the restriction of discrete series of a real reductive Lie group to a subgroup locally isomorphic to  $SL(2, \mathbb{R})$*

*Abstract:*

We consider a reductive real connected Lie group  $G$ , and a discrete series representation  $\rho$  of  $G$ . We determine the subgroups  $H$  of  $G$ , locally isomorphic to  $SL(2, \mathbb{R})$ , such that the restriction of  $\rho$  to  $H$  is admissible (this means that, as a representation of  $H$ , it is a –possibly infinite– direct sum of irreducible representations of  $H$  occurring with finite multiplicities).

This is a joint work with Esther Galina and Jorge A. Vargas.

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**Toshiyuki KOBAYASHI:**

*Branching problems and symmetry breaking operators*

*Abstract:*

Branching problems ask how irreducible representations of groups  $G$  decompose when restricted to subgroups  $G'$ . We present a program on branching problems, from abstract feature to concrete construction of symmetry breaking operators.

As an abstract feature, we provide a geometric criterion on the pair of reductive groups for the multiplicities of the branching laws to be always of finite (more strongly, uniformly bounded) by using analysis on (real) spherical varieties.

As a concrete construction of symmetry breaking operators (SBOs), we explain an idea of the  $F$ -method in constructing differential SBOs. Finally, we discuss some classification results of all non-local and local SBOs by an example.

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**Martin OLBRICH:**

*TBA*

*Abstract:*

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**Pavle PANDZIC:**

*A generalization of Dirac index*

*Abstract:*

The usual notion of Dirac index for Harish-Chandra modules makes sense only for equal rank groups. We propose a more general definition which works also in the unequal rank case, where it yields an invariant we call twisted Dirac index. We show that twisted Dirac indices are related to twisted characters in the same way as ordinary indices are related to ordinary characters. We compute indices for standard modules, we study the classification of twisted Cartan subgroups, and we give applications to extensions of modules. This is joint work with Dan Barbasch and Peter Trapa.

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**Tomasz PRZEBINDA:**

*The character and the wave front set correspondence in the stable range*

*Abstract:*

We relate the distribution characters and the wave front sets of unitary representation for real reductive dual pairs of type I in the stable range.

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**Michelle VERGNE:**

*Indice équivariant des opérateurs de Dirac*

*Résumé :*

Soit  $M$  une variété compacte muni d'une structure Spin invariante par l'action d'un groupe compact. Si  $L$  est un fibré en ligne équivariant sur  $M$ , nous donnons une formule géométrique pour les multiplicités de l'indice équivariant de l'opérateur de Dirac sur  $M$  twisté par  $L$ .

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**David VOGAN:**

*Langland parameters and finite-dimensional representations*

*Abstract:*

A standard way to describe the representation theory of reductive groups over local fields is to say that compact groups are easy, so we should try to understand representations in terms of their restrictions to compact subgroups. I will argue that compact groups are in fact *not* that easy, and that we should try to understand their representations as restrictions of representations of noncompact reductive groups, and therefore in terms of arithmetic and Langlands dual groups. I'll explain how this looks for maximal compact subgroups of real reductive groups (which are more or less understood) and for finite Chevalley groups (which George Lusztig understands, but almost nobody else does), and a little bit for  $p$ -adic groups (which nobody at all understands).

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**Genkai ZHANG:**

*Boundary value problem for some conformal invariant differential operators*

*Abstract:*

The complementary series for the rank one groups  $SO(n, 1)$  and  $SU(n, 1)$  have discrete components under the branching of their subgroups  $SO(n - 1, 1)$  and  $SU(n, 1)$  for appropriate parameters. This can be viewed as the trace of Sobolev spaces on  $\mathbb{R}^{n-1}$  restricted to  $\mathbb{R}^{n-2}$ . The dual operator of the restriction then solves certain Dirichlet or mixed Dirichlet problems. We prove certain  $L^p$ -boundedness properties for the solution operator of the boundary value problem.

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**Roger ZIERAU:**

*Characteristic cycles of highest weight Harish-Chandra modules*

*Abstract:*

This lecture will show how we may determine Harish-Chandra cells, associated varieties and characteristic cycles of highest weight Harish-Chandra modules for  $Sp(2n, \mathbb{R})$ . The main result is an algorithm that computes the characteristic cycle from the clan describing the support of the Harish-Chandra module.

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